# LOGISTIC AND RATIO MODELS FOR LIVESTOCK ESTIMATION

JOSE S. GUTIERREZ[1]

The series of values assumed by a variable at different points of time are not generally of the regular functional type in which the values can be represented exactly by a mathematical function of time. There is, however, good reason to believe that true trend of series connected with population is a logistic. Since in practice a large number of economic time series are more or less closely connected with population, it is likely that this type of economic series will approximately follow the path of human population growth or the modifications of this path.

## Logistic model of population growth

Rhodes (7) has defined the rate of increase of a population in a unit of time as the ratio of the increase, in the unit of time, of the population to the population at the beginning of this time interval. For example, if a population of size P at a time t increases by an amount dP at a corresponding time interval dt, the rate of increase is given by the following equation

$$R = \frac{1}{P}\frac{dP}{dt} \tag{1}$$

which could be written as

$$\log P = \int R \, dt + C \tag{2}$$

1 Assistant Professor of Statistics and Economics, U. P. Statistical Center

where C is the constant of integration. This equation gives the mathematical relation of population P and time t. The solution gives the mathematical relation of population P and time t. The solution however, of this equation depends upon the form of R. It is evident that there is no limit to the number of growth equations which may be derived from equation (2) by giving R different forms. The simplest case of these equations occurs when R is considered a constant. The growth equation will be of an exponential form

$$P = A\ e^{at} \qquad (3)$$

where A is a constant of integration. Theoretically, this might be a good growth model; however, this equation must be ruled out when studying long run tendencies. As Malthus has pointed out, considering the rate of increase as a constant is an absurdity (3).

A more appropriate assumption as regards R is to consider it as decreasing gradually as time t and population P, increase. The form which R could take even under this assumption will vary. It might be computationally practical to assume that the form in which R changes is a linear function of P, that is, R is equal to a(1 — aP). In this case, the differential equation for P becomes

$$\frac{1}{p}\frac{dP}{dt} = a\ (1 - aP) \qquad (4)$$

which on integration yields

$$P = \frac{\frac{1}{a}}{1 + \frac{1}{A}\,e^{-at}} \qquad (5)$$

where A is a constant of integration. By setting k = 1/a and b = 1/aA equation (5) becomes

$$P = \frac{k}{1 + b\,e^{-at}} \qquad (6)$$

Equation (6) is generally called the **logistic curve of population growth.**

**Estimation of the parameters of the logistic curve**

The difficulties to estimate the logistic fit from results of the logistic model of population growth are well-known. The application of the least square method or the maximum-likehood method yields normal equations with parameters entering in non-linear fashion and therefore connot be solved directly. Hotelling (3) had presented a very interesting method for the estimation of these parameters. He proposed the use of the differential equations and justify their use by a broad assumption. He said that in any problem the fundamental working assumption of the differential method is, not that a differential holds at all times and everywhere, but that the most probable value of the derivative at any instant is that assigned by the differential equation.

Tintner (9) considers the logistic as a law of population development and therefore the population density is proportional to the population. Then a simple transformation could be used:

$$Z_t = \frac{1}{P_t} = \frac{1 + b\,e^{-at}}{K}. \qquad (7)$$

Consider the value of Z in a period t + 1, then

$$Z_{t+1} = \frac{1 + b\,e^{-a(t+1)}}{k}$$

$$= \frac{1 + b\, e^{-at}\, e^{-al}}{k} \qquad (8)$$

From equation (7) the value of $b\, e^{-at}$ is $k\, Z_t - 1$ and substituting in equation (8)

$$Z_{t+1} = \frac{1 - e^{-al} + k\, e^{-al}\, Z_t}{k}$$

$$= \frac{1 - e^{-al}}{k} + e^{-al}\, Z_t \qquad (9)$$

Since the unit used is a one year period $l = 1$ then

$$Z_{t+1} = \frac{1 - e^{-a}}{k} + e^{-a}\, Z_t \qquad (10)$$

which is a simple linear difference equation with constant coefficients.

The simplicity and linearity of this model equation (10) could be readily seen if A is put for $\frac{1 - e^{-a}}{k}$ and $B = e^{-a}$, then the difference equation could be written as

$$Z_{t+1} = A + B\, Z_t \qquad (11)$$

The estimates A and B are obtained by least square method. The variances and covariances, $S^2$, $S^2$ , $S^2$ , $S_B$ are likewise estimated. The approximate variances of the estimates a, b, and k were obtained by the method of statistical differential.

## Application of the logistic model to livestock estimation

The feasibility of using the logistic model in livestock estimation in the Philippines has been examined in this study. The logistic models have been fitted to different livestock population for 1950-1964. The results of this fitting are as follows:

Carabao

$$Z_{t+1} = 2.8768 \times 10^{-7} + .9160\ Z_t \qquad (12)$$

$$(9.639 \times 10^{-8})\ (.2900)\ (2.30 \times 10^{-14})$$

$$s^2_A \qquad s^2_B \qquad s^2$$

Cattle

$$Z_{t+1} = 9.71 \times 10^{-7} + .9160\ Z_t \qquad (13)$$

$$(4.291 \times 10^{-7})\ (.3725)\ (1.90 \times 10^{-12})$$

Horses

$$Z_{t+1} = 2.9997 \times 10^{-6} + .35087\ Z_t \qquad (14)$$

$$(6.4319 \times 10^{-6})\ (1.36842)\ (7.80 \times 10^{-13})$$

Hogs

$$Z_{t+1} = .5\ 639 \times 10^{-8} + .6564\ Z_t \qquad (15)$$

$$(2.967 \times 10^{-8})\ (.15951)\ (2.60 \times 10^{-14})$$

Goats

$$Z_{t+1} = 8.128 \times 10^{-7} + .58897\ Z_t \qquad (16)$$

$$(1.1630 \times 10^{-6})\ (.54690)\ (1.131 \times 10^{-12})$$

Sheep

$$Z_{t+1} = 4.9753 \times 10^{-5} - 079487 \; Z_t \qquad (17)$$
$$(1.163 \times 10^{-6}) \; (.02107) \; (8.6475 \times 10^{-9})$$

Chicken

$$Z_{t+1} = 4\;247 \times 10^{-9} + .76285 \; Z_t \qquad (18)$$
$$(3.47 \times 10^{-8}) \; (.01446) \; (8.8 \times 10^{-18})$$

Ducks

$$Z_{t+1} = 1.497 \times 10^{-7} + .72353 \; Z_t \qquad (19)$$
$$(1.10 \times 10^{-8}) \; (.00152) \; (2.0 \times 10^{-14})$$

Geese

$$Z_{t+1} = 7.721 \times 10^{-7} + .48190 \; Z_t \qquad (20)$$
$$(8.824 \times 10^{-6}) \; (.48286) \; (1.0072 \times 10^{-9})$$

Turkey

$$Z_{t+1} = -1.244 \times 10^{-6} + 97037 \; Z_t \qquad (21)$$
$$(4.1211 \times 10^{-6}) \; (.14454) \; (1.786 \times 10^{-10})$$

The quantities below each model are $s^2_A$ ; $s^2_B$ and $s^2$ ;respectively.

Using the above logistic models, the population of each type of livestock was estimated in 1951 to 1965. The results of the estimation are presented in table 1. Significant results were obtained on the difference between the official and logistic estimates of the population of carabao, ducks, and geese.

## TABLE 1 ESTIMATES OF LIVESTOCK POPULATION IN THE PHILIPPINES 1950-65

| | Carabao | | Cattle | | Horses | |
|---|---|---|---|---|---|---|
| Year | Official | Logistic | Official | Logistic | Official | Logistic |
| 1950 | 1,902,920 | — | 698,060 | — | 206,140 | — |
| 1951 | 2,342,540 | 2,020,200 | 715,450 | 718,030 | 206,600 | 212,680 |
| 9152 | 2,439,070 | 2,471,150 | 738,990 | 737,190 | 213,580 | 212,860 |
| 1953 | 2,510,110 | 2,569,370 | 762,290 | 763,360 | 219,330 | 215,400 |
| 1954 | 2,980,590 | 2,635,190 | 763,350 | 789,140 | 197,200 | 217,200 |
| 1955 | 3,279,110 | 1,108,200 | 805,860 | 790,260 | 207,710 | 209,250 |
| 1956 | 3,594,680 | 3,403,210 | 861,160 | 837,800 | 218,420 | 213,270 |
| 1957 | 3,584,130 | 3,719,130 | 883,040 | 900,330 | 219,220 | 217,100 |
| 1958 | 3,596,390 | 3,714,570 | 896,270 | 25,240 | 220,900 | 217,380 |
| 1959 | 3,773,000 | 3,726,620 | 933,200 | 940,380 | 227,300 | 217,960 |
| 1960 | 3,696,300 | 3,899,850 | 1,110,500 | 982,800 | 217,400 | 220,960 |
| 1961 | 3,452,000 | 3,824,680 | 1,054,700 | 1,190,760 | 197,300 | 216,750 |
| 1962 | 3,471,800 | 3,584,230 | 1,094,400 | 1,124,450 | 210,000 | 209,290 |
| 1963 | 3,323,100 | 3,603,860 | 1,197,900 | 1,171,650 | 220,200 | 214,110 |
| 1964 | 3,100,700 | 3,456,740 | 1,382,900 | 1,296,340 | 242,100 | 217,720 |
| 1965 | | 3,235,620 | | 1,527,880 | | 224,770 |
| sd | 22,380 | | 57,732 | | 10,183 | |
| $\bar{d}$ | —42,390 | | 2,360 | | 3,240 | |
| \|d\| | 196,180 | | 37,950 | | 7,610 | |
| d | 5,983 | | 15,436 | | 2,722 | |
| F | 7.085 | | 0.1528 | | 1.190 | |
| P-Level | $P < .001$ | | $P > .50$ | | $.30 > P > .20$ | |

TABLE 1 (Continued)

ESTIMATES OF LIVESTOCK POPULATION IN THE PHILIPPINES 1950-65

| | Hogs | | Goats | | Sheep | |
|---|---|---|---|---|---|---|
| Year | Official | Logistic | Official | Logistic | Official | Logistic |
| 1950 | 3,899,130 | — | 355,430 | — | 26,350 | — |
| 1951 | 4,158,630 | 4,449,780 | 376,960 | 404,900 | 21,150 | 21,400 |
| 1952 | 4,442,540 | 4,667,880 | 384,000 | 421,020 | 21,760 | 20,840 |
| 1953 | 4,793,620 | 4,898,600 | 391,600 | 426,170 | 20,710 | 21,690 |
| 1954 | 4,867,630 | 5,172,770 | 438,200 | 431,220 | 15,720 | 21,780 |
| 1955 | 5,289,390 | 5,229,310 | 458,760 | 436,650 | 16,440 | 22,370 |
| 1956 | 5,749,880 | 5,540,780 | 497,850 | 476,960 | 17,150 | 22,260 |
| 1957 | 6,026,150 | 5,863,730 | 530,220 | 501,050 | 17,920 | 22,160 |
| 1958 | 6,083,620 | 6,049,240 | 537,060 | 519,860 | 16,560 | 22,070 |
| 1959 | 6,573,900 | 6,087,170 | 565,700 | 523,730 | 16,800 | 22,240 |
| 1960 | 6,572,600 | 6,400,410 | 617,100 | 539,730 | 14,800 | 21,780 |
| 1961 | 6,191,400 | 6,400,000 | 532,300 | 565,870 | 20,100 | 22,530 |
| 1962 | 6,725,700 | 6,157,260 | 628,300 | 521,010 | 22,500 | 21,850 |
| 1963 | 6,233,700 | 6,494,350 | 483,500 | 571,400 | 13,600 | 21,640 |
| 1964 | 6,616,400 | 6,185,060 | 557,500 | 492,220 | 4,400 | 22,770 |
| 1965 | | 6,427,150 | | 535,000 | | 20,860 |
| $s_d$ | 286,932 | | 50,665 | | | |
| $d$ | 59,200 | | 11,030 | | | |
| $\|d\|$ | 251,460 | | 43,580 | | | |
| $s_{\bar{d}}$ | 76,719 | | 13,546 | | | |
| F | 0.77 | | 0.880 | | | |
| P-level | .50 < P < .40 | | .40 < P < .30 | | .40 < P < .30 | |

## TABLE 1 (Continued)

## ESTIMATES OF LIVESTOCK POPULATION IN THE PHILIPPINES 1950-65

| | Chickens | | Ducks | | Geese | | Turkey | |
|---|---|---|---|---|---|---|---|---|
| | Offcial | Logistic | Official | Logistic | Offcial | Logistic | Official | Logistic |
| 1950 | 25,235,000 | — | 709,260 | — | 27,000 | — | 25,000 | — |
| 1951 | 28,054,900 | 29,005,090 | 752,290 | 854,850 | 79,600 | 86,890 | 93,000 | 1,187,510 |
| 1952 | 32,089,580 | 31,808,640 | 1,046,600 | 899,770 | 25,000 | 39,110 | 24,020 | 26,620 |
| 1953 | 37,392,150 | 35,690,070 | 1,243,900 | 1,189,060 | 30,000 | 37,040 | 24,900 | 25,540 |
| 1954 | 39,804,870 | 40,572,890 | 1,379,380 | 1,367,430 | 25,450 | 42,050 | 24,900 | 26,510 |
| 1955 | 44,583,590 | 42,714,500 | 1,695,550 | 1,483,240 | 85,370 | 37,520 | 30,210 | 26,726 |
| 1956 | 49,775,770 | 46,823,060 | 2,142,910 | 1,734,910 | 91,360 | 74,820 | 33,540 | 32,390 |
| 1957 | 51,838,700 | 51,093,400 | 2,077,780 | 2,052,120 | 95,980 | 76,950 | 37,010 | 36,120 |
| 1958 | 52,408,780 | 52,737,050 | 2,108,740 | 2,008,440 | 102,040 | 78,490 | 41,250 | 40,040 |
| 1959 | 56,141,500 | 53,185,830 | 2,097,200 | 2,029,220 | 97,969 | 82,350 | 41,430 | 44,880 |
| 1960 | 52,335,100 | 56,069,530 | 2,230,700 | 2,021,430 | 97,200 | 79,110 | 40,600 | 45,090 |
| 1961 | 49,984,400 | 53,126,490 | 1,784,100 | 2,109,700 | 97,200 | 78,870 | 38,200 | 44,140 |
| 1962 | 51,353,600 | 51,261,020 | 1,934,100 | 1,801,150 | 77,800 | 78,870 | 40,400 | 41,390 |
| 1963 | 48,624,000 | 52,353,280 | 1,594,300 | 1,909,130 | 84,500 | 71,870 | 72,900 | 43,910 |
| 1964 | 51,648,200 | 50,163,030 | 1,602,300 | 1,657,000 | 127,200 | 74,490 | 100,400 | 82,880 |
| 1965 | | 52,584,530 | | 1,663,060 | | 72,600 | | 1,088,260 |
| sd | | 2,170,484 | | 19,240 | | 19,588 | | 281,683 |
| $\bar{d}$ | | +69,050 | | 40,890 | | 13,440 | | —75,790 |
| $\lvert\bar{d}\rvert$ | | 1,766,870 | | 154,840 | | 19,320 | | 83,390 |
| $\tilde{d}$ | | 580,340 | | 5,134 | | 5,237 | | 75,316 |
| F | | 0.118 | | 7.964 | | 2.566 | | 1.006 |
| | | $P > .50$ | | $P < .001$ | | $.02 > P > .05$ | | $.40 > P > .30$ |

**Ratio estimation of livestock population**

Suppose a variable reacts to the movements of another variable i.e. following the same type of probability distribution functions. The representation of such similarity in movements may be exploited in studying the movement of one based on the known movement of the other. A possible representation of such similarity in movements is

$$Z'_{t+1} = \frac{Z'_t}{Z_t} Z_{t+1} \qquad (22)$$

where the primes indicate the variable whose movement is being studied based on the known movement of the other variable. A more general representation of such similarity is

$$Z'_{t+1} = \frac{f(Z'_t)}{g(Z_t)} h(Z_{t+1}) \qquad (23)$$

where f, g, and h denote functions of $Z'_t$, $Z_t$ and $Z_{t+1}$ respectively.

It can be readily seen that equation (23) can be as simple as equation (22) or can be highly complicated depending upon the form assigned to f, g, and h.

Consider the simple relation given by equation (22). The unbiasedness of $\hat{Z}'_{t+1}$ as an estimator of $Z'_{t+1}$ depends on the assumption made on $Z'_{t+1}/Z_t$: If the simple lagged relation exists between $Z'_{t+1}$ and $Z'_t$, this can be formulated as follows:

$$Z'_{t+1} = w' Z'_t \qquad (24)$$

where w' is estimated by $\frac{Z_{t+1}}{Z_t}$. If the expectation $E(\frac{Z_{t+1}}{Z_t}) = w'$

then

$$\hat{Z}'_{t+1} = w' \hat{Z}_t \quad (25)$$

is an unbiased estimator of $Z'_{t+1}$. This unbiasedness of $\hat{Z}_{t+1}$ further assumes that $Z'_t$ and w' are independent.

The assumption $E(\frac{Z_{t+1}}{Z_t}) = w'$ implies that the rates of increase for the pairs of livestock considered are the same. For pairs such as carabao and cattle; sheep goats, and ducks and geese, this assumption is realistic, on account of the similarity in gestation or incubation periods of the pairs compared. It may however not be realistic for such pairs as carabao and hogs, cattle and sheep, and carabao and chicken due to the difference in gestation and incubation periods. Shorter gestation and incubation periods mean faster rates of increase.

With the latter comparison $\hat{Z}'_{t+1}$ may be unbiased estimator of $Z'_{t+1}$ if function of $Z_{t+1}/Z_t$ can be formulated such that

$$E[f(\frac{Z_{t+1}}{Z_t})] = w' \quad (26)$$

It may be noted that equations (24) and (25) are special forms of equation (11); equations (24) and (25) are really equation (11) with A equal to zero. The regression coefficient B is estimated by w' through the utilization of the known movements of a similarly distributed variable.

In this paper, carabao is set as the variable whose movement is known and other livestock including poultry as the variable whose movements are being studied given the carabao growth pattern.

The choice of carabao as the independent livestock is due to the pattern of Philippine agriculture. Philippine agriculture is basically one of rice. Farm surveys designed utilizing palay farm characteristics may yield unbiased estimators of farms carabao population but may result to biased estimates of the population of the livestocks (including poultry). The high degree of correlation between farm reporting rice production and farm reporting carabao might explain the unbiased etimator of carabao population. On the other hand, the relatively low correlation or no correlation at all between farms reporting other livestock might be the reason for the biased estimators of the population of this livestock. The relatively high correlation between farms reporting carabao and farms reporting livestock might suggest the feasibility of utilizing the principles of ratio estimation and similarly distributed variables to achieve an unbiased estimators with non-significant bias) of the population of these livestock. (This discussion is based on a preliminary correlation analysis done in the Bureau of Agricultural Economics).

The variance of $Z'_{t+1}$ is

$$\text{Var}\,(Z'_{t+1}) = E\left(\frac{Z_{t+1}}{Z_t}\right)^2 \text{Var}\,Z'_t + (E\,Z'_t)^2\,\text{Var}\left(\frac{Z_{t+1}}{Z_t}\right)$$

$$+ 2E\left(\frac{Z_{t+1}}{Z_t}\right) E\,(Z'_t)\,\text{Cov}\left(\frac{Z_{t+1}}{Z_t}, Z'_t\right) \qquad (27)$$

where

$$\text{Var}\,\frac{Z_{t+1}}{Z_t} = \frac{1}{(E\,Z_t)^2}[\text{Var}\,Z_{t+1} + E\,(\frac{Z_{t+1}}{Z_t})^2\,\text{Var}\,Z_t$$

$$-\,2E\,(\frac{Z_{t+1}}{Z_t})\,\text{Cov}\,)Z_t, Z_{t+1})] \qquad (28)$$

The estimator of Var $(Z'_{t+1})$ is

$$\text{Var}\,(Z'_{t+1}) = (\frac{\hat{Z}_{t+1}}{\hat{Z}_t})^2\,\text{Var}\,Z'_t + (Z'_t)^2\,\text{Var}\,(\frac{\hat{Z}_{t+1}}{\hat{Z}_t})$$

$$+\,2\,\frac{Z_{t+1}}{\hat{Z}_t}\,\hat{Z}'_t\,\text{Cov}\,(\frac{Z_{t+1}}{\hat{Z}_t}, \hat{Z}'_t) \qquad (29)$$

The values of $Z'_t/Z_t$ by year and by type of livestock for 1950 to 1964 are presented in table 2. A good precision in the sense of closeness of one $Z'_t/Z_t$ to another may also be noted. This closeness of one $Z'_t/Z_t$ to another justify the formulation of the equation (22). The estimated population values of different livestock using equations (22) and (24) are refered to as ratio estimates (table 3).

It can also be seen in the same table that the differences between the official estimates and ratio estimates of the population of different types of livestock do not differ significantly. There is however a tendency for the ratio estimates of the population of horses and sheeps to exceed those of the official estimmates. The ratio estimates of population number of other livestock are smaller than their corresponding official estimates. The absolute mean differences are presented in the aforementioned table.

TABLE 2

VALUES OF $Z_t'/Z_t$ BY YEAR AND BY TYPE OF LIVESTOCK

1950-1964

| Year | Cattle | Horses | Hogs | Goats | Sheep | Chicken | Ducks | Geese | Turkey |
|---|---|---|---|---|---|---|---|---|---|
| 1950 | .3668 | .1083 | 2.0490 | .1868 | .0138 | 13.2610 | .3727 | .0142 | .0131 |
| 1951 | .3054 | .0882 | 1.7753 | .1609 | .0090 | 11.9766 | .3212 | .0043 | .0043 |
| 1952 | .3020 | .0876 | 1.8214 | .1574 | .0089 | 13.1564 | .4291 | .0123 | .0102 |
| 1953 | .3044 | .0876 | 1.9145 | .1562 | .0083 | 14.9337 | .4968 | .0102 | .0099 |
| 1954 | .2567 | .0663 | 1.6371 | .1474 | .0053 | 13.3875 | .4638 | .0287 | .0102 |
| 1955 | .2465 | .0635 | 1.6179 | .1403 | .0050 | 13.6372 | .5186 | .0279 | .0103 |
| 1956 | .2400 | .0609 | 1.6021 | .1387 | .0048 | 13.8695 | .5971 | .0267 | .0103 |
| 1957 | .2464 | .0612 | 1.6813 | .1479 | .0050 | 14.4630 | .5797 | .0285 | .0115 |
| 1958 | .2492 | .0614 | 1.6916 | .1493 | .0046 | 14.5723 | .5863 | .0272 | .0115 |
| 1959 | .2473 | .0602 | 1.7421 | .1499 | .0045 | 14.8797 | .5558 | .0258 | .0108 |
| 1960 | .3004 | .0588 | 1.5491 | .1669 | .0040 | 14.1587 | .6035 | .0263 | .0103 |
| 1961 | .3055 | .0572 | 1.7935 | .1542 | .0058 | 14.4795 | .5168 | .0225 | .0211 |
| 1962 | .3152 | .0634 | 1.7955 | .1393 | .0039 | 14.4791 | .5571 | .0243 | .0210 |
| 1963 | .3603 | .0663 | 1.8758 | .1455 | .0041 | 14.6319 | .4798 | .0383 | .0302 |
| 1964 | .4460 | .0781 | 2.1338 | .1798 | .0014 | 16.6565 | .5167 | .0257 | .0300 |

# TABLE 3

## RATIO ESTIMATES OF LIVESTOCK POPULATION IN THE PHILIPPINES 1951-1964

| | Cattle | Horses | Hogs | Goats | Sheep | Chicken | Ducks | Geese | Turkey |
|---|---|---|---|---|---|---|---|---|---|
| 1951 | 859,240 | 253,700 | 4,799,860 | 437,590 | 32,330 | 41,064,420 | 873,060 | 33,260 | 30,690 |
| 1952 | 736,600 | 215,130 | 4,330,080 | 392,450 | 21,950 | 29,211,770 | 783,430 | 10,490 | 10,490 |
| 1953 | 758,050 | 219,890 | 4,571,910 | 395,090 | 22,340 | 33,024,010 | 1,077,090 | 30,870 | 25,600 |
| 1954 | 907,290 | 261,100 | 5,706,340 | 465,570 | 22,740 | 44,511,240 | 1,480,760 | 30,400 | 29,510 |
| 1955 | 841,750 | 217,400 | 5,368,230 | 483,340 | 17,380 | 43,899,090 | 1,521,180 | 94,110 | 33,450 |
| 1956 | 886,090 | 228,260 | 5,815,830 | 504,330 | 17,970 | 49,021,370 | 1,864,200 | 100,290 | 37,030 |
| 1957 | 860,190 | 218,270 | 5,742,130 | 497,120 | 17,200 | 49,710,090 | 2,140,080 | 95,700 | 36,560 |
| 1958 | 886,150 | 220,100 | 6,046,610 | 531,910 | 17,980 | 52,014,690 | 2,084,830 | 102,500 | 41,360 |
| 1959 | 940,230 | 231,660 | 6,382,310 | 563,310 | 17,360 | 54,980,290 | 2,212,110 | 102,630 | 43,390 |
| 1960 | 914,100 | 222,520 | 6,439,320 | 554,080 | 16,630 | 54,999,840 | 2,054,400 | 95,360 | 39,920 |
| 1961 | 1,036,980 | 202,980 | 5,347,490 | 576,140 | 13,810 | 48,875,830 | 2,083,280 | 90,790 | 35,560 |
| 1962 | 1,060,630 | 198,590 | 6,226,670 | 535,350 | 20,140 | 50,269,930 | 1,794,230 | 78,120 | 73,260 |
| 1963 | 1,047,440 | 210,680 | 5,966,630 | 462,910 | 12,960 | 48,115,500 | 1,851,300 | 80,750 | 69,790 |
| 1964 | 1,117,180 | 205,580 | 5,816,290 | 451,150 | 12,710 | 45,369,130 | 1,487,720 | 118,760 | 93,640 |
| $s_d$ | 82,867 | 22,742 | 443,396 | 40,620 | 4,500 | 2,754,796 | 185,823 | 22,869 | 8,067 |
| $s_{\bar{d}}$ | 22,156 | 6,080 | 118,550 | 10,860 | 1,203 | 734,612 | 49,683 | 6,114 | 2,156 |
| $\bar{d}$ | 7,790 | —7,960 | 112,551 | 13,470 | —5,170 | 783,420 | 27,300 | 3,760 | 3,000 |
| $\lvert\bar{d}\rvert$ | 58,590 | 14,790 | 344,655 | 33,310 | 5,390 | 2,266,370 | 163,810 | 15,600 | 4,230 |
| F | 0.3576 | 1.3092 | 0.9493 | 1.2403 | 1.0616 | 1.0664 | .5494 | 0.6149 | 1.3911 |

An interesting result of the ratio estimation livestock population number is the ratio estimates for poultry and smaller four-footed livestock e.g., sheep, goats, and hogs. As mentioned earlier the difference in gestation and incubation periods may result to difference in rates of increase between that of carabaos and those of other livestock. The results of this estimation seemed not to support this contention. The principal reason for this rather interesting result is the assumption that livestock number for period t + 1 is a function of the number for period t. This justify the lagged relation given by equation (24).

**Estimation of livestock population using ratio estimator with an inflator—deflator multiplier**

To improve the ratio estimation of a livestock number based on the known increase of another livestock, and inflator—déflator factor is introduced into the estimator. For this study a simple ratio of prices (values) of the livestock are used. In estimating the livestock number for period t + 1 the inflator—deflator factor used is the ratio of values for periods t and t — 1 ($P_t/P_{t-1}$) (see table 4). For smaller livestock (chicken and hogs) this ratio might reflect the price response of the animal husbandmen. However, for larger livestock (e.g., carabao and cattle) these periods used might be too short to give ample time for adjustment in the animal production plans.

## TABLE 4

### VALUES OF INDEF FACTOR $(P'_t/P'_{t-1})$ BY YEAR AND BY TYPE OF LIVESTOCK 1952 — 1965

| Year | Cattle | Horses | Hogs | Goats | Sheep | Chicken | Ducks | Geese | Turkey |
|---|---|---|---|---|---|---|---|---|---|
| 1952 | 1.3550 | 1.0133 | 1.0725 | .9700 | .9814 | .9784 | 1.2115 | 1.3734 | 1.1192 |
| 1953 | 1.0921 | 1.0250 | 1.0718 | .9515 | .8814 | 1.1323 | 1.0582 | .7500 | .9271 |
| 1954 | .9905 | 1.0028 | .9913 | .9973 | .9181 | .9610 | .9550 | 1.2125 | 1.1350 |
| 1955 | .8640 | .9475 | .5392 | .9577 | 1.0084 | .8513 | .8272 | 1.0618 | .9698 |
| 1956 | 1.0080 | 1.0127 | .9771 | .9954 | 1.0158 | .9761 | 1.0126 | 1.0000 | .9542 |
| 1957 | .9989 | 1.0024 | 1.0250 | .9889 | .9770 | 1.0162 | 1.0437 | 1.1521 | 1.0614 |
| 1958 | .9896 | 1.0067 | .9626 | .9859 | 1.0497 | .9760 | 1.0179 | .9466 | 1.0108 |
| 1959 | .9645 | .9734 | 1.0185 | .9563 | 1.2797 | 1.0245 | .9823 | .9910 | .9910 |
| 1960 | .9967 | 1.0054 | .9957 | 1.0009 | 1.0139 | .9920 | 1.0179 | 1.0179 | 1.0054 |
| 1961 | .9173 | .9297 | 1.1788 | 1.0069 | .8071 | 1.0403 | .9823 | .9647 | .9766 |
| 1962 | 1.3257 | 1.4991 | 1.2800 | .9911 | .8967 | 1.1550 | 1.0419 | .8506 | .7941 |
| 1963 | .9359 | 1.1468 | 1.0318 | 1.0515 | 1.0428 | .8590 | .8908 | .9856 | 1.0370 |
| 1964 | 1.1468 | .8049 | 1.0040 | 1.0933 | 1.1788 | 1.2109 | 1.2967 | 1.0472 | 1.3571 |
| 1965 | 1.0310 | 1.0100 | 1.0016 | .9810 | 1.1292 | 1.0193 | .9150 | 1.2951 | 1.0246 |

The differences between the official estimates of livestock number and those obtained using ratio estimator with "indif" (inflator—deflator) factor are not statistically significant (table 5). Larger livestock number are obtained for cattle, horses, and sheeps and the rest of the estimates are all smaller than the corresponding official estimates.

The average differences and the average absolute differences are generally larger than those corresponding values obtained using simple ratio estimators.

The results of this aspect of the study seem to strengthen the assumption on expressing the livestock number for a period as a function of the number for the preceeding period. For short period projections this assumption may hold, however, for long period projections there may still be a need to use more sophisticated indef models say, regression models.

**Comparison of the different estimators**

The values of the standard error of differences, standard error of mean differences, mean, differences and mean absolute mean differences and estimated F-values, and the corresponding probability levels are given in table 6.

Two significant differences were obtained for comparison, official (0) and logistic (L) and one each for comparison ratio (R) and ratio with indef factor (RI) and logistic and ratio with indef factor.

# TABLE 5

## ESTIMATES OF LIVESTOCK POPULATION IN THE PHILIPPINES 1952—1964 USING RATIO ESTIMATOR WITH AN INFLATOR—DEFLATOR MULTIPLIER

| | Cattle | Horses | Hogs | Goats | Sheep | Chicken | Ducks | Geese | Turkey |
|---|---|---|---|---|---|---|---|---|---|
| 1952 | 998,090 | 217,990 | 4,644,010 | 380,680 | 21,540 | 28,580,800 | 949,130 | 14,410 | 11,740 |
| 1953 | 827,870 | 225,400 | 4,900,170 | 375,930 | 19,690 | 37,393,090 | 1,139,780 | 23,150 | 23,730 |
| 1954 | 898,700 | 261,830 | 5,656,690 | 464,310 | 22,710 | 42,775,300 | 1,414,130 | 36,860 | 34,490 |
| 1955 | 727,300 | 206,010 | 2,894,550* | 462,890 | 17,530 | 37,712,950 | 1,258,320 | 99,930 | 32,440 |
| 1956 | 893,220 | 231,170 | 5,682,650 | 502,010 | 18,250 | 47,849,760 | 1,889,690 | 100,290 | 35,330 |
| 1957 | 859,310 | 218,790 | 5,885,680 | 491,600 | 16,800 | 50,515,390 | 2,233,600 | 110,260 | 38,800 |
| 1958 | 876,980 | 221,590 | 5,820,470 | 524,410 | 18,870 | 50,766,340 | 2,122,150 | 97,030 | 41,810 |
| 1959 | 906,850 | 225,050 | 6,500,380 | 538,690 | 22,220 | 56,327,310 | 2,172,960 | 101,710 | 43,000 |
| 1960 | 911,660 | 223,420 | 6,411,630 | 554,640 | 16,860 | 54,559,840 | 2,091,170 | 97,070 | 40,140 |
| 1961 | 951,180 | 188,710 | 6,303,620 | 580,110 | 11,150 | 50,845,530 | 2,046,410 | 87,590 | 34,730 |
| 1962 | 1,406,130 | 297,730 | 7,970,140 | 530,590 | 18,060 | 58,061,770 | 1,869,410 | 66,450 | 58,180 |
| 1963 | 980,040 | 241,620 | 6,156,370 | 486,750 | 13,510 | 41,331,210 | 1,649,140 | 79,590 | 72,370 |
| 1964 | 1,281,180 | 165,480 | 5,839,560 | 493,240 | 15,280 | 50,047,960 | 1,929,130 | 124,370 | 127,080 |
| $s_d$ | 152,676 | 38,323 | 810,123 | 38,157 | 4,762 | 3,650,671 | 194,602 | 23,628 | 13,360 |
| $\bar{d}$ | 42,291 | 10,587 | 216,602 | 10,202 | 1,319 | 1,014,075 | 53,807 | 6,544 | 3,700 |
| $\bar{d}$ | —41,000 | —8,780 | 115,430 | 18,128 | -1,070 | 864,080 | 37,420 | 4,070 | 2,580 |
| $\|d\|$ | 121,050 | 22,560 | 492.930 | 31.276 | 3,510 | 2,853,490 | 155,560 | 16,410 | 8,540 |
| F | 0.9695 | 0.8293 | 0.5329 | 1.7769 | 0.8112 | 0.8520 | 0.6954 | 0.6219 | 0.6972 |

*The writer has some reservation as regard the reported value of hogs particularly those for fiscal year 1954 to 1961.

# TABLE 6

## VALUES OF $^{s}d$, $^{s}\bar{d}$, $\bar{d}$, $|\bar{d}|$, F, AND P—LEVEL FOR VARIOUS ESTIMATORS OF LIVESTOCK NUMBERS

| CATTLE | O — L | O — R | O—RI | L — R | L—RI | R—RI |
|---|---|---|---|---|---|---|
| $^{s}d$ | 57,732 | 82,867 | 152,676 | 74,417 | 138,057 | 129,112 |
| $^{s}\bar{d}$ | 15,435 | 22,156 | 42,291 | 19,896 | 38,241 | 35,764 |
| $\bar{d}$ | 2,360 | 7,790 | —41,000 | —960 | —7,490 | —22,060 |
| $\lvert\bar{d}\rvert$ | 37,950 | 58,500 | 121,050 | 54,490 | 112,550 | 83,160 |
| F | 0.1528 | 0.3516 | 0.9695 | 0.0482 | 0.1958 | 0.6168 |
| HORSES | | | | | | |
| $^{s}d$ | 10,183 | 22,742 | 38,223 | 16,012 | 31,326 | 28,824 |
| $^{s}\bar{d}$ | 2,722 | 6,080 | 10,587 | 4,281 | 8,677 | 7,984 |
| $\bar{d}$ | 3,240 | —7,960 | —8,780 | —7,200 | —7,580 | —11,380 |
| $\lvert\bar{d}\rvert$ | 7,610 | 14,790 | 22,560 | 12,430 | 21,640 | 16,360 |
| F | 1.190 | 1.3092 | 0.8293 | 1.6818 | 0.8736 | 1.4253 |

## TABLE 6

### VALUES OF $^{s}d$, $^{s}\bar{d}$, $\bar{d}$, $|\bar{d}|$, F, AND P—LEVEL FOR VARIOUS ESTIMATORS .. (CONT.)



| HOGS | O — L | O — R | O — RI | L — R | L — RI | R — RI |
|---|---|---|---|---|---|---|
| $^{s}d$ | 286,932 | 443,396 | 810,123 | 386,005 | 841,169 | 501,957 |
| $^{s}\bar{d}$ | 76,719 | 118,550 | 216,602 | 106,923 | 233,003 | 139,042 |
| $\bar{d}$ | 59,200 | 112,551 | 115,430 | 133,930 | 1,778 | 300,467 |
| $\overline{\lvert d\rvert}$ | 251,469 | 344,655 | 492,930 | 303,504 | 481,067 | 367,646 |
| F | 0.7716 | 0.9493 | 0.5329 | 1.2525 | 0.0076 | 2.1609 |

| GOATS | O — L | O — R | O — RI | L — R | L — RI | R — RI |
|---|---|---|---|---|---|---|
| $^{s}d$ | 50,665 | 40,620 | 38,157 | 40,124 | 39,458 | 17,421 |
| $^{s}\bar{d}$ | 13,546 | 10,860 | 10,202 | 10,727 | 10,929 | 4,825 |
| $\bar{d}$ | 11,930 | 13,470 | 18,128 | —1,348 | —1,457 | —2,070 |
| $\overline{\lvert d\rvert}$ | 43,580 | 33,310 | 31,276 | 31,797 | 28,383 | 12,910 |
| F | 0.8800 | 1.2403 | 1.7769 | 0.1256 | 0.1333 | 0.4290 |

| SHEEP | O—L | O—R | O—RI | | | |
|---|---|---|---|---|---|---|
| $s_d$ | 18,215 | 4,500 | 4,762 | 53,113 | 32,863 | 2044 |
| $s_{\bar{d}}$ | 4,870 | 1,203 | 1,319 | 14,200 | 9,102 | 568 |
| $\bar{d}$ | —5,170 | —1,860 | —1,070 | —2,990 | 4,180 | 700 |
| $\lvert\bar{d}\rvert$ | 5,390 | 3,280 | 3,510 | | | |
| F | 1.0616 | 1.5461 | 0.8112 | 0.2105 | 0.4591 | 1.2323 |

| CHICKENS | | | | | | |
|---|---|---|---|---|---|---|
| $s_d$ | 2,170,484 | 2,754,796 | 3,650,671 | 2,633,818 | 5,218,834 | 3,903,255 |
| $s_{\bar{d}}$ | 580,340 | 73,612 | 1,014,075 | ,702,351 | 1,445,660 | 1,084,238 |
| $\bar{d}$ | 69,050 | 783,420 | 864,080 | 824,090 | 987,780 | —211,110 |
| $\lvert\bar{d}\rvert$ | 1,766,890 | 2,266,370 | 2,853,490 | 2,420,440 | 4,326,940 | 3,010,700 |
| F | 0.1180 | 1.0664 | 0.8520 | 1.1733 | 0.6832 | 0.1947 |

# TABLE 6

## VALUES OF $^{s}d$, $^{s}d$, d, ldl, F, AND P—LEVEL FOR VARIOUS ESTIMATORS . . . (CONT.)

| | | | | | | |
|---|---|---|---|---|---|---|
| DUKS | | | | | | |
| $\|d\|$ | 154,840 | 163,810 | 155,560 | 98,442 | 152,577 | 164,742 |
| $^{s}d$ | 19,240 | 185,823 | 194,602 | 26,320 | 42,121 | 45,633 |
| $^{s}\bar{d}$ | 5,134 | 49,683 | 53,807 | —13,590 | 3,370 | 25,200 |
| $\bar{d}$ | 40,890 | 27,300 | 37,420 | 83,410 | 130,410 | 118,750 |
| $\overline{\|d\|}$ | 154,840 | 163,810 | 155,560 | 0.5163 | 0.0800 | 0.5533 |
| F | 749641 | .5494 | 0.6954 | 15,649 | 17,874 | 5,954 |
| GEESE | | | | | | |
| $^{s}d$ | 19,588 | 22,869 | 23,628 | 4,184 | 4,951 | 1,649 |
| $^{s}\bar{d}$ | 5,237 | 6,114 | 6,544 | —8,970 | —10,720 | 2,930 |
| $\bar{d}$ | 13,440 | 3,760 | 4,070 | 16,210 | 17,460 | 5,230 |
| $\overline{\|d\|}$ | 19,320 | 15,600 | 16,410 | 2.1438 | 2.1652 | 1.7768 |
| TURKEY | | | | | | |
| $^{s}d$ | 281,683 | 8,067 | 13,360 | 280,481 | 281,134 | 9,693 |
| $^{s}\bar{d}$ | 75,316 | 2,156 | 3,700 | 74,992 | 77,874 | 2,684 |
| $\bar{d}$ | —75,790 | 3,000 | 2,580 | 83,650 | 83,330 | 3,620 |
| $\overline{\|d\|}$ | 83,390 | 4,230 | 8,540 | | | |
| F | 1.0060 | 1.3911 | 0.6972 | 1.1154 | 1.070 | 1.3789 |

The lack of information on the variances of the official estimates make it impossible to compare the variances of the different estimators against those of the official.

## Summary and conclusion

The feasibility of estimating (projection) the population number of a given livestock for a certain period based on information available for another livestock has been demonstrated in this study. Lagged relationships have been utilized in ratio projection of livestock numbers. The use of inflator—deflator factors have also been investigated. The lack of the estimated variances for the general estimates of livestock number prevented the stuudy from making investigations on the relative efficiency of the different estimators considered. However, differences between estimates when treated statistically seemed to favor the feasibility of projecting a livestock number using a livestock number using the known number of another livestock.

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